



Investigation Of Seasonal Variation and Trend Effect in Stochastic Model Selection

Mehmet Ali HINIS

Aksaray Üniversitesi Mühendislik Fakültesi, İnşaat Mühendisliği Bölümü, AKSARAY

*Sorumlu Yazar
mhinis@gmail.com

Abstract: Stochastic models are frequently used methods in hydrology. These models are used to produce longer time period synthetic data having the same statistical parameters of limited observed data. Hydrologic data have usually seasonal variation and trend factors which effect to determine the best fit stochastic model.

The effects of these parameters on various stochastic models were investigated in this study by using monthly averaged flow data of Demirci (16-080) flow observation station in Konya basin in the present study. In various orders of AR, ARMA, SAR and SARMA models were evaluated for different seasonal effect and trend factor. In the second part of the study, several criteria were compared to determine the best fit model.

Keywords: Stochastic model, AR, ARMA, SAR, SARMA, trend and seasonal effect, estimation and diagnostic checking

INTRODUCTION

Stochastic models are one of the statistical methods often used in hydrological analysis to generate synthetic data based on observations and to make predictions by using the generated data. Hydrological data are used as the basis of water resources planning and design projects. To use long term observational data is important to increase the reliability of planning. In order to get the necessary information and data, modeling of streamflow is important for a variety of applications in engineering structures for example water resources planning, water supply and management of irrigation systems and water quality, drought analysis etc.

Statistical methods are used to model the time dependent variables that affect the hydrological parameters. Main purposes for the modeling of variables following the stochastic process are to generate statistically similar synthetic data to observed data and to forecast future generating similar synthetic data assuming past conditions will not change. The stochastically generated data is not the future or past observed data but predicted data of similar conditions with observed data [1]. There are many studies on the stochastically generated hydrological data in the literature [2], [3], [4], [5] and [6]. Among many methods, autoregressive, moving average, exponential smoothing, Holt's method could be given for stochastically forecasting. Holt's method was weak in capable of seasonal fluctuation and exponential method was weak in trend capturing in time series data. Seasonal fluctuation, periodic behavior and trend effect are the most important features of an observed hydrological data. ARIMA and Winter method are used to predict the data with these features.

Hydrological time series due to their periodic recurrences can be estimated by linear stochastic models. Linear stochastic models are generally known as Auto Regressive Integrated Moving Average (ARIMA) or Box and Jenkins models. Estimation capacity of ARIMA models are reported to have more flexibility than other models such as neural networks, fuzzy system due to its regressive and moving average features [1]. Several autoregressive and moving average models are developed to predict the seasonal and periodic fluctuations. ARIMA model was applied on monitoring agricultural drought by [10] and the performance of the model was found rather good. SARIMA model was used by [11] to forecast the drought in Kansabati river basin in India and it was concluded that the model gave reasonably good results for 2 month lead forecasting and it can be used for drought preparedness plans in the region. Streamflow of four rivers selected from various places in the world was simulated by [12] by using ARMA models and prediction interval of the models are reported as better than bootstrap methods.

30 years of monthly flow volumes of Demirci (16-080) Flow Observation Station is used in this study. First 25 years of the flow data (600 months) is modeled in various orders of AR, ARIMA and SARIMA models and various criteria are used to select the best fit model. Winters method, s one of the best methods to model the seasonal fluctuations, is also used to model the streamflow data. The results of Winter method and SARIMA method are compared. In the second part of the study, 5 years of data (60 months) are forecasted by using both methods and results are compared with the real observed data.

METHOD

The use of Auto Regressive Integrated Moving Average (ARIMA) model and how to make short terms predictions in a time series are explained in [13]. The model is mathematically defined in terms of residuals which are based on statistically the differences in observed and predicted values. The time series is constructed equally spaced data since it can be used to explain the seasonal fluctuations. The model with minimum residual is taken as the best fit model. Stochastic processes modeling is used in generation by using the statistical properties of observed data. Forecasting is the next step after selecting the best fit model. Daily, monthly and seasonal data are generally used in time series analysis. As time periods decreases, internal dependency and irregularity in the data emerges. For example, since time series of yearly data has less correlation than that of daily data, it can be defined with less order models. AR(1) is the least order of the model and it can be used to define short term dependency, however higher orders may be needed to explain the long term relations and seasonal ARIMA (SARIMA) modes are preferred in seasonal or periodic relations.

The first step in modeling of a time series is to separate the deterministic and stochastic components. Deterministic components can be seen as periodic recurrence, jump and trend in a hydrologic series which makes the time series non-stationary. Stochastic components are statistical terms which define the autoregressive and random changes in the series [14]. Streamflow series can be modeled in four steps in hydrology:

1. Model selection: The best model type and order is selected to define the observed series. There are some guidelines to select the model and order type based on the auto correlation and partial autocorrelation relations (correlograms) but it is usually selected based on the experience.
2. Defining model parameters: There are several methods to find the model parameters such as moments, maximum likelihood, least square methods. Model parameters are estimated based on the observed data.
3. Diagnostic checking: Tentative model is checked for adequacy with observed data. There are several criteria for this purposes such as: residual autocorrelation function (ACF), AIC (Akaike Information criteria), SBIC (Schwarz Bayesian Information Criteria), minimum error variance, Portmanteau test (Ljung-Box test). There are some weakness in each criteria, thus, different results of several criteria are compared for the best fit model selection [15].
4. Generation of synthetic series: Selected model is used to generate the short term forecasting.

The models and formulations used in this study are explained briefly in the following parts.

Linear Autoregressive Models (AR)

Flow in year t with p order autoregressive model is given as:

$$x_t = \mu + \sum_{j=1}^p \phi_j (x_{t-j} - \mu) + \varepsilon_t \quad (1)$$

Where p ; model order, x_t ; flow in year t , ε_t ; residual term with zero mean and variance s_ε^2 , ϕ_j ; autoregressive term

Linear autoregressive and moving average models (ARMA)

This model consist two models emerged; p order of AR and q order of MA (moving average) terms. General structure of this model is given below;

$$x_t = \mu + \sum_{j=1}^p \phi_j (x_{t-j} - \mu) + \varepsilon_t - \sum_{j=1}^q \theta_j \varepsilon_{t-j} \quad (2.a)$$

Where θ_j ; moving average parameter. Based on the model parameter orders, it can be shown as ARMA(p,q). In explicit form it is given as

$$x_t = \phi_1 x_{t-1} + \dots + \phi_p x_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q}, \quad (2.b)$$

Backshift operator (B) is suggested to ease the use of model. For example: $Bx_t = x_{t-1}$ represents the flow in time $t-1$. Therefore ARMA model can be expressed the following;

$$(1 - \phi_1 B - \dots - \phi_p B^p)x_t = (1 - \theta_1 B - \dots - \theta_q B^q)e_t \quad (3)$$

$$\phi_p(B)x_t = \theta_q(B)e_t \quad (4)$$

If a time series is not stationary, it can be made stationary buy differencing [13] as shown in the following equation;

$$(1-B)x_t = x_t - x_{t-1} \quad (5)$$

Thus the model is called ARIMA (auto regressive integrated moving average). In practical application, differencing one time is usually enough to make the series stationary.

Linear seasonal autoregressive moving average models (Seasonal ARIMA models)

Seasonal ARIMA (SARIMA) models are recommended in modeling of seasonally fluctuated time series [13]. The model is given in general form as;

$$\phi_p(B)\Phi_p(B)(1-B)^d(1-B^s)^D x_t = \theta_q(B)\Theta_q(B^s)\varepsilon_t \quad (6a)$$

And in implicit form;

$$(1 - \phi_1 B - \dots - \phi_p B^p) (1 - \Phi_1 B^s - \dots - \Phi_p B^{sp}) (1-B)^d (1-B^s)^D X_t = C + (1 + \theta_1 B + \dots + \theta_q B^q) (1 - \Theta_1 B^s - \dots - \Theta_q B^{sq}) \varepsilon_t \quad (6b)$$

where d ; number of differencing, s ; number of season, D ; number of seasonal differencing, Φ_p ; q^{th} order autoregressive term, θ_p ; q^{th} order of seasonal autoregressive term, Θ_q ; q^{th} order of moving average term, Θ_q ; q^{th} order of seasonal moving average term. SARIMA models can be written as SARIMA(p,d,q)(P,D,Q) s based on the orders of AR(p) and MA(q) processes and the presence of differencing (d)

Best fit model is selected among many candidates based on the statistical criteria and residual term analysis. Model selection is explained in selection criteria section. The parsimony is adopted in the selection of model that based on the preference of least parameter model.

Holt’s Winter exponential smoothing method

This method is suggested for slow changing and seasonally fluctuated time series modeling. Based on the constant and variable correction term, there are additive and multiplicative exponential smoothing methods respectively.

Additive seasonal Holt’s-Winter exponentially smoothing method is given below;

$$a_t = \alpha(x_t - F_{t-s}) + (1-\alpha)(a_{t-1} + b_{t-1}) \tag{7a}$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)b_{t-1} \tag{7b}$$

$$F_t = \gamma(x_t - a_t) + (1-\gamma)F_{t-s} \tag{7c}$$

Where α, β, γ ; smoothing constants whose values are changing within zero and one, a_t level in time t , b_t ; slope at time t , s ; number of period in a year, x_t ; observed time series at time t .

Multiplicative seasonal Holt’s-Winter exponentially smoothing method is given below;

$$a_t = \alpha(x_t / F_{t-s}) + (1-\alpha)(a_{t-1} + b_{t-1}) \tag{8a}$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)b_{t-1} \tag{8b}$$

$$F_t = \gamma(x_t / a_t) + (1-\gamma)F_{t-s} \tag{8c}$$

Model Selection Criteria

Once the stochastic model is selected, model parameters are checked against reversibility and normality test. After residual values are tested for normality and independent and identically distributed there are many tests in the literature to select the best fit model to define the stochastic process. The most common technique is to examine the autocorrelation (ACF) and partial autocorrelation function (PACF). Assumed random and independent residual terms should be below pre-defined confidence interval (usually 95%). Port Manteau test is another common method used to check if the residuals are random and independent [2]. It is given as;

$$Q = n(n+2) \sum_{k=1}^m (n-k)^{-1} r_k(\epsilon)^2 \tag{9}$$

where Q statistic is compared with χ^2 (chi square) statistic with $(m-n-q)$ degrees of freedom in a given significance level, m ; lag time, $r_k(\epsilon)$ residual correlogram, n ; number of observation. If there is any differencing (d) then n is replaced with n' where $n' = n - d$. If probability of Q is less than 0.01%, then the model is not adequate, if it is bigger than 5% then the selected model is adequate. Correlogram of residuals is given in equation (10). The tested null hypothesis is that the current set of autocorrelations is white noise.

$$r_k(\epsilon) = \frac{\sum_{t=1}^{n-k} \epsilon_t \epsilon_{t+k}}{\sum_{t=1}^n \epsilon_t^2} \tag{10}$$

There are other test methods to choose the best fit among candidate models in the literature based on the residual terms. Most common methods are given briefly in the following section.

Sum of Square Errors, (SSE);

$$SSE = \sum_{t=1}^n (y_t - \hat{y}_t)^2 \tag{11}$$

where y_t ; observed data, \hat{y}_t ; estimated data.

Mean Square Error, (MSE);

$$MSE = \frac{SSE}{n} \tag{12}$$

Akaike Information Criteria (AIC): One of the most common test methods in stochastic analysis is that AIC criteria suggested by [16] based on the least variable usage in the model (the parsimony criteria). AIC is given as;

$$AIC = n \ln(MSE) + 2k \tag{13}$$

k ; sum of AR and MA parameters.

Schwarz Bayesian Information Criterion, (SBIC);

Similar to AIC, [17] suggested the equation below;

$$SBIC = n \ln(MSE) + k \ln(n) \tag{14}$$

Amemiya’s Prediction Criterion, (APC)[18];

$$APC = \left(\frac{n+k}{n-k} \right) \frac{SSE}{n} \tag{15}$$

Amemiya’s Adjusted R-Square;

$$AAR = 1 - \left(\frac{n+k}{n-k} \right) (1 - R^2) \tag{16}$$

where R^2 is correlation based on observed and estimated data and given below;

$$R^2 = 1 - \frac{\sum_{t=1}^n (y_t - \hat{y}_t)^2}{\sum_{t=1}^n (y_t - \bar{y})^2} \tag{17}$$

DATA

Monthly flow volumes of Karasu river located in Demirci Flow Observation Station (16-080) in Aksaray province is used. Data spans 1970-2000 period. The station has 1118 metre elevation and 497 m² rainfall basin. The model structure has two parts: the first part (1970-1995) has 300 monthly flow values and used in the selection of best model. Based on the selected model 5 year (60 months) of flow values are forecasted. The second part of the data (1996-2000) which has 60 monthly flow values is used in comparison of forecasted values.

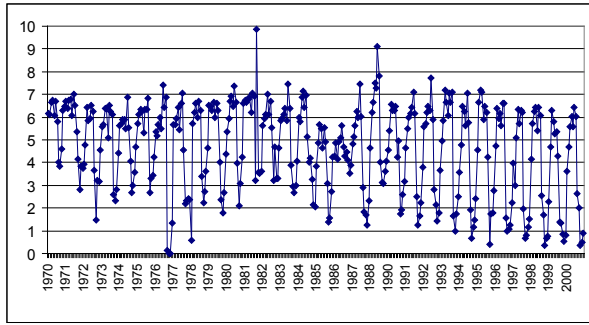


Figure 1. Karasu river monthly flow data (1970-2000)

Karasu river flow data is given in Figure 1 and its correlograms of ACF and PACF along with their 95% confidence intervals ($\pm 0,011547$) are given Figure 2.

Figure 2 shows that ACF is different than zero and there is periodicity in every 12 lag. ACF and PACF correlograms indicate that there are AR and MA functions affecting the flow time series and there is a periodicity which should not be ignored. Based on Figure 2, alternative stochastic models are searched and results are evaluated in the following section.

RESULTS AND DISCUSSION

A linear trend with a negative slope of $-0,0031$ and cutting point of $5,3257$ in observed flow series was found. Hypothesis test was constructed whether slope is significant than zero and it was checked with non-parametric Spearman rho test. t-test value was found as -2.5162 and probability of 1.24% which indicates that the slope is significant. Therefore the trend is included in the analysis.

Several AR, MA, ARMA, ARIMA and SARIMA models were generated to find the best fit to observed data series according to ACF and PACF correlograms given in Figure 2. Based on the best fit criteria given in section 2, results are evaluated and given in Table 1, Figure 3.a and Figure 3.b. Models are generated by subtracted mean and including trend of the observed data. Seasonal effect is taken as 12 in SARMA models and significant level was considered as 5% in the statistical evaluations. Model evaluation is carried out on the results of accepted models which pass the ACF and PACF correlograms of residuals. Once the models pass these criteria, then Port-Manteau

Table 1. Searched models and best fit criteria in stochastic model selection

Model no	Model	R ²	SSE	MSE	RMSE	P.T.	K.K.	AIC	SBIC	APC	AAR
1	AR(4)	0.579	436.185	1.474	1.214	N	N	126.851	145.353	1.528	0.558
2	ARMA(4,3)	0.684	326.683	1.115	1.056	N	N	121.418	151.021	1.501	0.566
3	AR(12,0,0)	0.685	326.265	1.133	1.064	N	N	55.840	103.947	1.205	0.651
4	ARMA(11,1,0)	0.675	336.901	1.170	1.082	N	N	60.547	104.952	1.225	0.646
5	ARMA(12,0,2)	0.686	324.585	1.135	1.065	N	N	58.825	114.331	1.218	0.648
6	ARMA(12,0,4)	0.687	324.294	1.142	1.069	N	N	62.486	125.394	1.233	0.643
7	ARMA(12,1,2)	0.683	327.900	1.151	1.073	N	N	70.065	121.918	1.200	0.415
8	ARMA(12,1,4)	0.681	330.293	1.167	1.080	N	N	78.360	137.621	1.225	0.403
9	SARMA(1,0,0)(0,0,1)	0.564	458.397	1.515	1.231	N	N	128.568	135.976	1.548	0.309
10	SARMA(1,0,0)(1,0,0)	0.626	387.031	1.299	1.140	N	N	84.198	95.299	1.325	0.617
11	SARMA(1,0,1)(2,0,0)	0.678	332.970	1.125	1.061	N	N	44.020	62.521	1.158	0.665
12	SARMA(4,0,3)(2,0,0)	0.687	324.478	1.082	1.040	N	N	45.531	86.273	1.164	0.663
13	SARMA(4,1,3)(2,0,0)	0.682	328.377	1.098	1.048	N	N	46.022	79.326	1.166	0.663
14	SARIMA(0,0,0)(2,1,0)	0.614	390.787	1.357	1.165	N	N	91.898	99.225	1.376	0.609
15	SARIMA(2,1,0)(2,1,0)	0.638	365.536	1.238	1.129	N	N	77.421	92.059	1.310	0.627
16	SARIMA(1,0,1)(1,1,1)	0.714	289.745	1.006	1.003	N	N	9.739	24.391	1.034	0.706
17	SARMA(0,0,0)(1,0,1)	0.625	387.721	1.301	1.141	N	N	82.957	90.365	1.310	0.383
18	SARMA(0,0,1)(1,0,1)	0.689	322.297	1.085	1.042	N	N	30.522	41.634	1.096	0.464
19	SARMA(0,0,2)(1,0,1)	0.705	305.390	1.032	1.016	Y	Y	17.369	32.184	1.045	0.483
20	SARMA(1,0,0)(1,0,1)	0.712	297.954	1.003	1.002	Y	Y	6.963	18.074	1.013	0.497
21	SARMA(1,0,1)(1,0,1)	0.703	307.693	1.039	1.019	Y	Y	19.623	34.438	1.053	0.480
22	SARMA(1,0,2)(1,0,1)	0.705	305.472	1.035	1.018	Y	Y	58.618	77.120	1.217	0.648
23	SARIMA(2,0,1)(2,1,0)	0.700	303.383	1.053	1.026	Y	Y	24.986	43.301	1.091	0.690
24	SARMA(2,0,2)(1,0,1)	0.708	302.815	1.030	1.015	Y	Y	20.863	43.086	1.051	0.480
25	SARMA(2,0,2)(2,0,1)	0.710	299.830	1.023	1.012	Y	Y	20.912	46.839	1.047	0.481
26	SARMA(3,0,3)(1,0,1)	0.704	306.065	1.048	1.024	Y	Y	30.113	59.743	1.076	0.469
27	SARMA(3,0,3)(2,0,1)	0.707	303.619	1.043	1.021	Y	Y	30.735	64.069	1.075	0.468
28	SARIMA(2,0,0)(2,1,0)	0.700	303.995	1.056	1.027	Y	Y	23.566	38.218	1.085	0.691
29	SARMA(4,0,3)(2,0,1)	0.715	294.916	1.017	1.008	Y	Y	25.043	62.081	1.051	0.478
30	SARMA(4,0,3)(2,1,0)	0.703	300.816	1.045	1.022	Y	Y	30.538	63.505	1.111	0.694
31	W. M. A.	0.711	298.879	0.996	0.998			4.877	15.988	1.016	0.705
32	W. M. M.	0.667	1035.300	1.148	1.071			47.299	58.41	1.17078	0.665

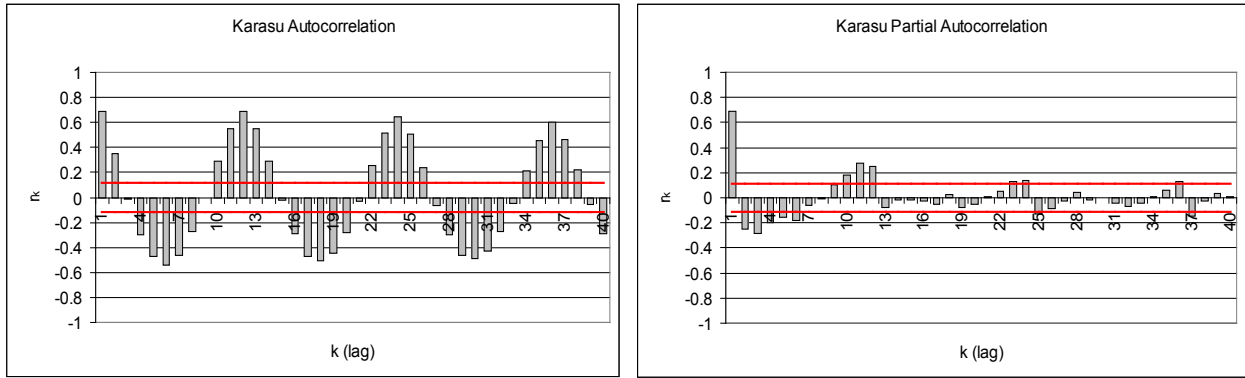


Figure 2. ACF (left) and PACF (right) correlograms for the period of 1970-1996

test (Q test) is applied. After the Q test is achieved by the model, the other criteria given in section 2 is applied and performances are compared to find the best fit model.

Abbreviations used in Table 1 are as the following: P.T.; Port-Manteau Test, R.C.; Residual Correlogram, W.M.A.; Holt’s Winter Additive method, W.M.M.: Holt’s Winter Multiplicative method, Y: yes (passed the test), N: No (not passed the test) and the rest is as given before.

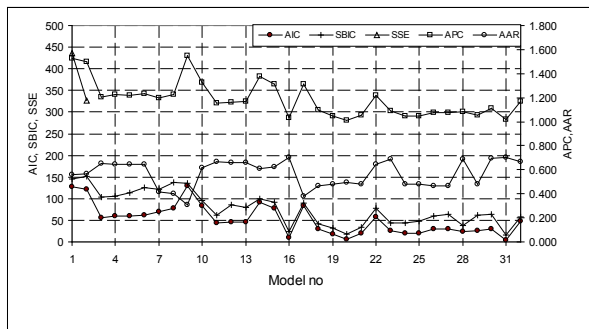


Figure 3.a. AIC, SBIC, SSE, APC and AAR criteria for all models

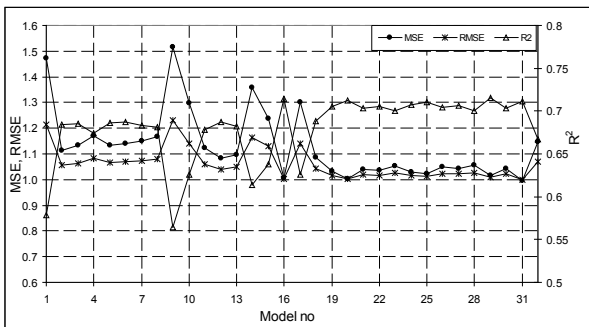


Figure 3.b. MSE, RSE and R2 criteria for all models.

There are 14 models (Model 19 through Model 32) which pass the Port-Manteau and within the ACF/PACF confidence limit. AIC, SBIC and APC criteria were changing in the same level for different models. Figure 3.a. and Figure 3.b. show that Model 20 (SARMA (1,0,0) (1,0,1)) and for Model 31 (Holt’s-Winter Additive model) were the best fit models.

Parameters of these models were given in Table 2. Since the b parameter is close to zero value, this can be omitted. Stream flow values calculated with these models are compared with observed data and given in

Figure 4 and Figure 5 for SARMA and W.M.A. models respectively.

60 months (5 year) of stream flow for 1996-2000 period were forecasted with the selected models and compared with observed (real) data and results are given in Figure 6 and Figure 7.

It can be clearly seen from Figure 6 and Figure 7 that forecasted values are closely related with observed values.

Statistical analyses were carried out between observed flow and forecasted values from both methods and results are given in Table 3 and Table 4. Statistical analysis revealed that forecasted values obtained from both methods (SARMA and Holt’s-Winter Additive methods) are very good estimates of the observed values. Durbin Watson test showed that there is a significant serial correlation between observed and forecasted values. Residual terms are tested for normality and found with constant variance and fit normal distribution.

Table 2. Parameters of the selected models

SARMA(100)(101)	ϕ_1	0.46704
	Φ_1	0.99908
	Θ_1	0.779502
Holt’s Winter Additive (WMA)	α	0.183617
	β	6.45E-08
	γ	0.144225

Table 3. Basic statistics of forecasted flows in between 1996-2000 period

	Mean	Standart deviation	Standart Error	Skewness	Curtosis
Observed Flow	3.692	2.296	0.296	-0.149	1.336
SARMA(100)(101)	4.553	1.941	0.251	-0.353	1.568
WMA	4.246	1.757	0.227	-0.364	1.561

Table 4. Linear regression results of observed and forecasted flow for 1999-2000 period

Flow	Durbin-Watson	Linear Correlation	R ²	R	MSE
Observed & SARMA	1.7021	Yes	0.883	0.94	0.626
Observed & WMA	1.5254	Yes	0.8859	0.9412	0.611

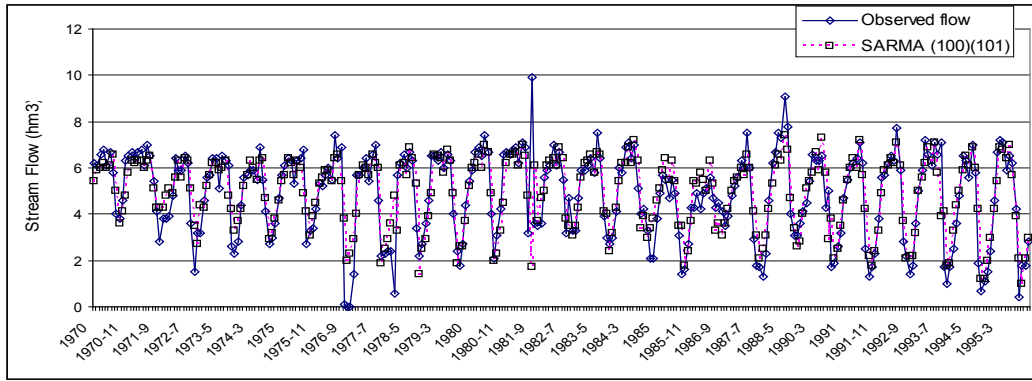


Figure 4. SARMA(100)(101) model and observed data

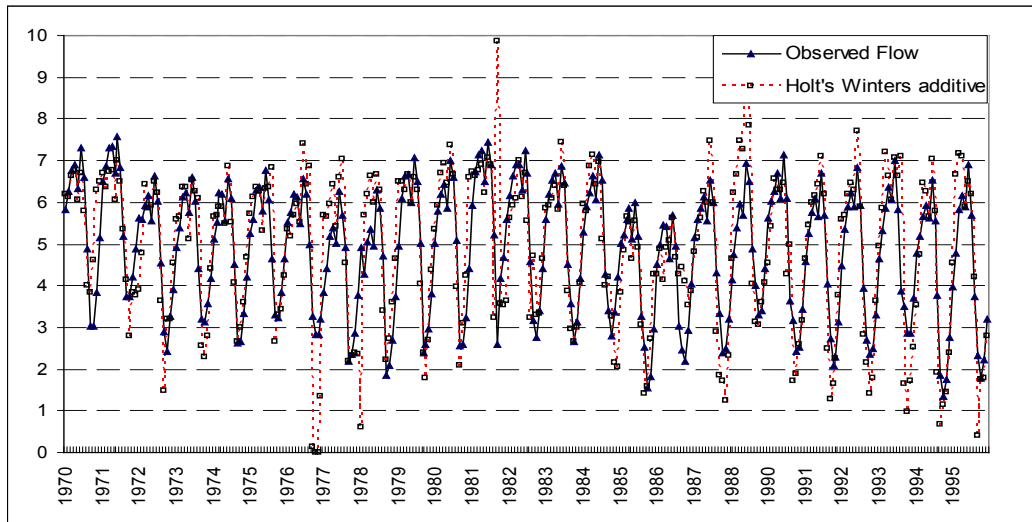


Figure 5. Holt's Winter additive model and observed data

CONCLUSION

Monthly stream flow values of Demirci (16-080) Flow Observation Station data near Aksaray city in Konya Closed basin is simulated by constructing various stochastic models. Data spans for 1970-2000 period (300 months) and best fit model is found as SARMA(100)(101) (Seasonal ARIMA) which reflects seasonal effects of data. Trend analysis is carried out in data and linear decreasing equation was found significant and its effect was included in the developed models. Seasonal

fluctuation was also considered in the models. After selecting the best fit model, 5 year (60 months) of data was forecasted and compared with observed values. Comparison revealed that the developed model was in good agreement with observed data.

Monthly flow values were also modeled with Holt's-Winter additive method and it was also found in good agreement. SARMA and Holt's-Winter additive model were compared and MSE and R2 values were found very close to each other resulting that both models reflects the observed flow conditions very well.

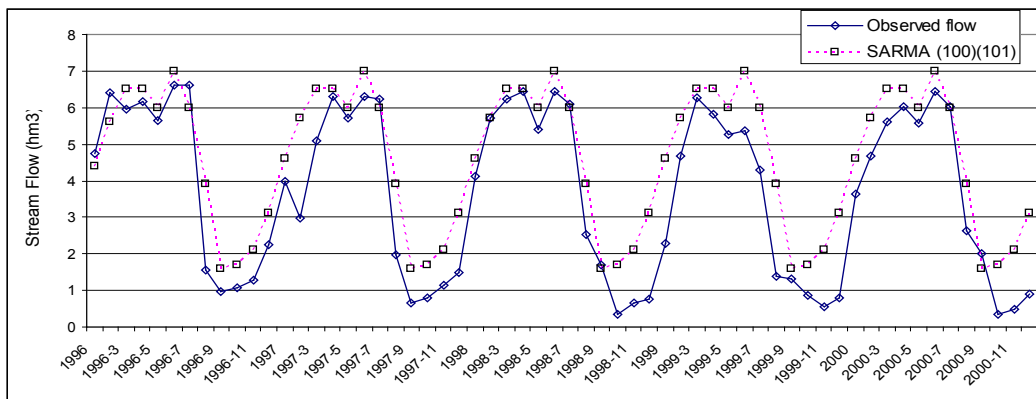


Figure 6. Comparison of forecasted values with SARMA(100)(101) model and Observed flow

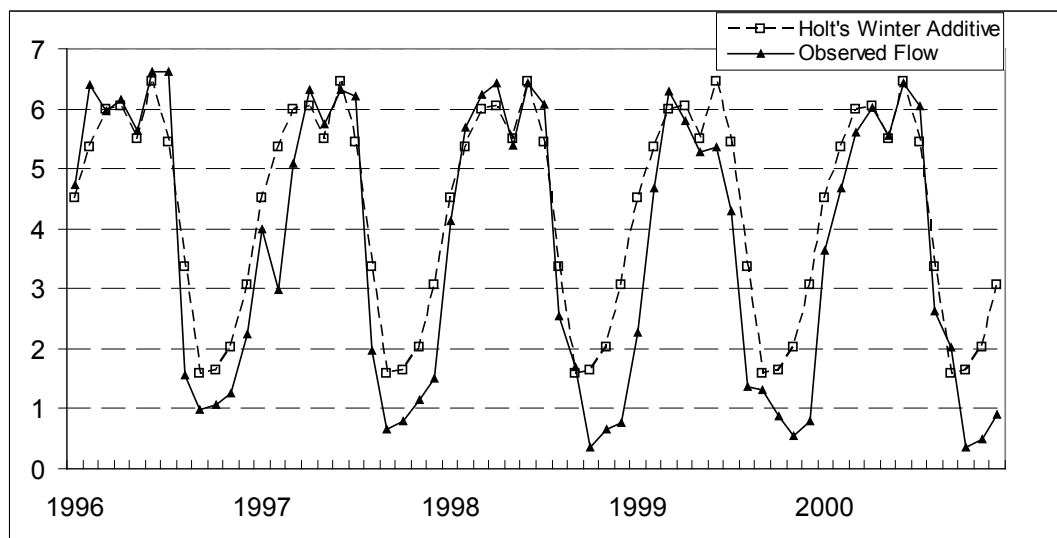


Figure 7. Comparison of forecasted values with Holt's Winter Additive model and Observed flow

REFERENCES

- [1] Yurekli, K., Kurunç, A., Ozturk, F., 2005, Application of linear stochastic models to monthly flow data of Kelkit stream, *Ecological Modelling*, 183(1), 67-75.
- [2] Salas J.D., Delleur, J.W., Yevjevich, V., Lane, W.L., 1980, *Applied Modelling of Hydrologic Time Series*, Water Resources Publications, Littleton, Colorado.
- [3] Hirsch, R.M., Helsel, D.R., Cohn, T.A., Gilroy, E.J., 1993, *Statistical analysis of hydrologic data*, in; *Handbook of Hydrology* edited by, D.R. Maidment, 19.1-19.72, Mc-Graw-Hill, New-York.
- [4] Salas, J.D., 1993, *Analysis and Modelling of hydrologic time series*, in *Handbook of Hydrology*, edited by, D.R. Maidment, 19.1-19.72, Mc-Graw-Hill, New-York.
- [5] Yevjevich, V., 1972, *Stochastic processes in Hydrology*, Water Resources Publications, Colorado.
- [6] Yevjevich, V. 1984, *Structure of Daily Hydrologic Series*, Water Resources Publications, Colorado.
- [7] Makridakes S, Wheelwright S, Hyndman R. 1998. *Forecasting Methods and Applications*. New York: John Wiley & Sons.
- [8] Salas J.D., Pielke R., 2003, *Stochastic Characteristics and Modeling of Hydroclimatic Processes* in *Handbook of Weather, Climate, and Water: Atmospheric Chemistry, Hydrology, and Societal Impacts*, Edited by Thomas D. Potter and Brndley R. Colman. ISBN 0-471-21489-2 John Wiley & Sons, Inc.
- [9] Wang, W., Pieter, H.A.J.M., Gelder, V., Vrijling, J.K., 2005, *Constructing prediction intervals for monthly streamflow forecasts*, ISSH, *Stochastic Hydraulics*, Nijmegen, Netherlands.
- [10] Yurekli, K., Kurunç, A., 2006, *Simulating agricultural drought periods based on daily rainfall and crop water consumption*, *J. Arid Environments*, 67, 629-640.
- [11] Mishra, A.K., Desai, V.R., 2005, *Drought forecasting using stochastic models*, *Stochastic Env. Res.. Risk Asses.*, 19: 326-339.
- [12] Wang, W., Pieter, H. A. J. M., Gelder, V., Vrijling, J. K., J. M., 2005. *Testing and modelling autoregressive conditional heteroskedasticity of streamflow processes*. *Nonlinear Processes in Geophysics* 12: 55-66
- [13] Box G.E.P., Jenkins, G.M., 1976, *Time Series Analysis: forecasting and control*, Holden-Day, Merrifield, Virginia.
- [14] Bayazit, M., 1996, *Probabilistic Methods in Civil Engineering* (In Turkish), İTÜ publ.
- [15] Baran, Bacanlı 2006, *Evaluation of Suitable Stochastic Model Criteria*, (In Turkish) İMO Teknik Dergi (Chamber of Civil Engineering Technical Journal), 2006 3987-4002, Paper ID 264
- [16] Akaike H., 1974, *A New Look at the Statistical Model Identification*, *IEEE Transactions On Automatic Control*, Vol. ac-19, Ko. 6, December.
- [17] Schwarz, G., 1978, *Estimating the dimension of a model*, *Annals of Statist.* Vol: 6, (2) 461-464.
- [18] Amemiya, T. (1980), "Selection of Regressors," *International Economic Review*, 21, 331-354.